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Forecasting the Time Series of Wolf Numbers for the 23rd Solar Cycle

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Abstract—A parametric autoregression model is constructed for the time series of Wolf numbers for the current period. Recursion calculations are used to obtain the predicted values of the series of W_t for the 23rd solar cycle and their 95%-probability errors. It is shown that the forthcoming cycle will most probably be low, that the predicted values do not vary linearly, and that the maximum of their variations, 73.8 ± 10, will occur in 2001.

INTRODUCTION

One of the fundamental problems in studies of the astronomical time series of Wolf numbers (W_t) is modeling it by parametric methods. Parametric models for various problems related to time series are mainly used for forecasting and automatic control.

Presently, the expected value of the forthcoming 23rd solar cycle is being discussed in the astronomical literature with animation. A review of these papers can be found in Obridko (1995). Studies based on the Gnevyshev–Ol' rule yield high values for the cycle maximum (up to 210–225). More sophisticated methods, which make use of geomagnetic data, give low values (~70). However, these methods can give a reliable predicted value not earlier than 1997.

The aim of this paper is to parametrically model the time series of Wolf numbers for the current period using the theory and practical techniques developed by Box and Jenkins (1970) and Bondar' *et al.* (1995) and to obtain numerical estimates of the Wolf numbers for the 23rd solar cycle.

NUMERICAL CALCULATIONS: ALGORITHM AND RESULTS

We briefly summarize the forecasting technique developed by Box and Jenkins (1970) which we have put to practical use (Bondar' *et al.* 1995).

For forecasting, we use a parametric model that is described by the difference equation

$$\begin{aligned} Z_{t+1} &= \varphi_1 Z_{t+l-1} + \varphi_2 Z_{t+l-2} + \ldots + \varphi_{p+d} Z_{t+l-p-d} \\ &\quad - \theta_1 a_{t+l-1} - \theta_2 a_{t+l-2} - \ldots - \theta_q a_{t+l-q} + a_{t+l}, \end{aligned} \quad (1)$$

where Z_t are the numerical values of the time series of W_t; a_t is a random variable of white noise; φ_p and θ_q are the parameters of the autoregression model and the moving average, respectively; and $l \geq 1$.

Based on formula (1), we can derive the predicted values $Z_t(l)$ of the series Z_{t+1}. In other words, the forecast is made at time t with a lead l. In this case, the variance of the forecast error for l steps for any time t, defined as the expectation value of $L_t^2(t) = [Z_{t+l} - \hat{Z}_t(l)]^2$, is equal to

$$V_l = \left\{1 + \sum_{j=1}^{t-1} \psi_j^2\right\}\sigma_a^2, \quad (2)$$

where σ_a^2 is the white-noise error and ψ_j are the weights derived from solution of the equation

$$\varphi(B)(1 + \psi_1 B + \psi_2 B^2 + \ldots) = \theta(B). \quad (3)$$

If the number of initial data points is not less than 50 σ_a^2, the unconditional sum of squares S_a^2 can be used to determine $S_a^2 = S(\bar{\varphi}, \bar{\theta})/n$. σ_a in formula (2) is then replaced by S_a, and the approximate $(1 - \xi)$%-probability limits for Z_{t+l} take the form

$$Z_{t+l} = Z_t(l) \pm U_{\xi/2}\left\{1 + \sum_{j=1}^{l-1} \psi_j^2\right\}^{1/2} S_a, \quad (4)$$

where $U_{\xi/2}$ is the quantile of level $(1 - \xi/2)$ of the standard normal distribution.

Thus, if we have information about the time series by the time t, then the predicted numerical values will lie with a probability of $(1 - \xi)$ within the range given by formula (4). Note that the probability values refer only to individual forecasts with different leads.

To make a forecast for the 23rd solar cycle, we took the time series of Wolf numbers W_t for the period from 1890 to 1994 with the sampling step $t = 1$ year. We will

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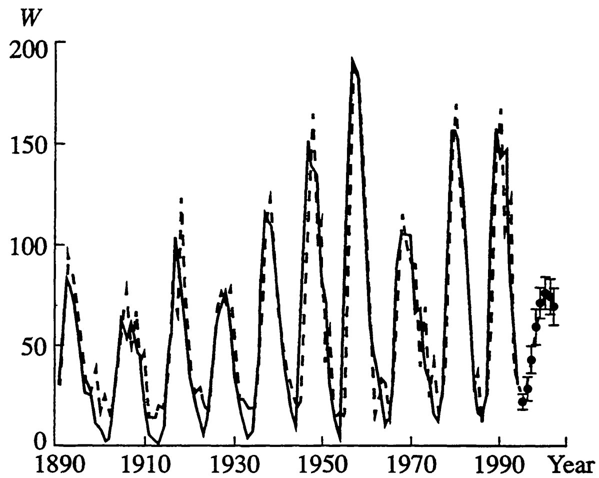

Fig. 1. Variations in the Wolf numbers and a parametric model of these variations for an eight-year forecast.

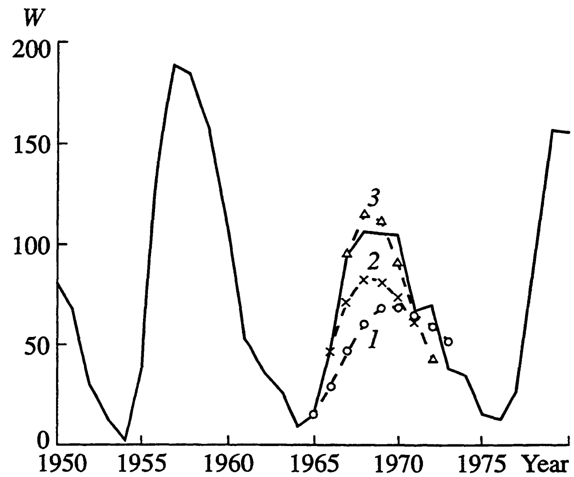

Fig. 2. Predicted values of Wolf numbers derived from different points of the series.

not dwell on the technique of parametric modeling, but turn to a description of the forecasting results based on formulas (1)–(4). In Fig. 1, variations in the Wolf numbers over the time interval from 1890 to 1994 are indicated by the solid line, and a parametric fit to the series of W_t is indicated by the dashed line. The circles in this figure show the predicted values of Z_{t+l} at the current time t (1994) with lead l, where l was varied from one to eight years. The predicted values of $Z_t(l)$ are given with their 95% confidence intervals. The rms errors are ±4.8 for a one-year forecast, ±10.2 for a five-year forecast, and ±18.6 for an eight-year forecast. Analysis of our calculations shows that the technique used for stochastic parametric modeling yields values of the Wolf numbers for the 23rd solar cycle that are lower than for the preceding 22nd solar cycle by a factor of almost 2 at maximum (2001). The slope of the curve changes after 1997; it is for this period that we can give an optimal value for W_t at the maximum of the 23rd solar cycle.

Considering the foregoing discussion, the natural question arises as to whether our results and the forecasting method based on parametric autoregression modeling are realistic. For this purpose, we made a forecast for the 20th solar cycle, which is known to be low in the value of Wolf numbers. As in the previous case, we took the time series of W_t for the period from 1890 to 1965. The results of our modeling and the initial time series of W_t for the 19th and 20th cycles are shown in Fig. 2. Curves *1*, *2*, and *3* in this figure indicate the predicted values. Curve *1* was obtained by calculating the predicted values beginning in 1965 with lead l, where l was varied from one to eight years; curve *2* gives the initial point of the forecast calculation for 1966; finally, curve *3* gives the forecast calculated beginning in 1967. In all three cases, the forecast for one year is the best. For any of the initial points of the series of W_t, the error is ±5 for the one-year forecast and ±13 for the five-year forecast.

We can see that in all three cases the forecasting technique employed did not yield large values of W_t similar to those of the 19th cycle. This gives us hope that our calculations of the time series of W_t for the 23rd cycle are realistic.

CONCLUSION

In conclusion, let us formulate the main result of this study. We have constructed a parametric autoregression model for the time series of Wolf numbers for the current period. Using simple recursion calculations, we have obtained the predicted values of the series of W_t for the 23rd solar cycle and their 95%-probability errors. We show that the forthcoming cycle will most probably be low, that the predicted values do not vary linearly, and that the maximum of their variations, 73.8 ± 10, will occur in 2001. This implies that Gnevyshev–Ol' rule will be violated in the 23rd cycle. A more accurate value for the maximum can be given in 1997.

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